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**Search for  $\eta'_c$  and  $h_c(^1P_1)$  states  
in the  $e^+e^-$  annihilations**

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**Abstract**

Productions and decays of spin-singlet  $S, P$ –wave charmonium states,  $\eta'_c$  and  $h_c(^1P_1)$ , in the  $e^+e^-$  annihilations are considered in the QCD multipole expansion with neglecting nonlocality in time coming from the color-octet intermediate states. Our approximation is opposite to the Kuang-Yan’s model. The results are  $B(\psi' \rightarrow h_c + \pi^0) \approx 0.3\%$ ,  $B(\psi' \rightarrow \eta' + \gamma) \approx 0.34\%$ ,  $\Gamma(\eta'_c \rightarrow J/\psi + \gamma) \approx 0.26$  keV and  $\Gamma(h_c \rightarrow J/\psi + \pi^0) \approx 2.5$  keV.

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Among the heavy quarkonium states, the spin-singlet  $P$ –wave state ( $h_{c,b}(^1P_1)$ ) is extremely difficult to be produced in the  $e^+e^-$  annihilations, because it has  $J^{PC} = 1^{+-}$ . Such a state in the charmonium family was first discovered in  $p\bar{p}$  annihilation by E760 Collaboration at Fermi Lab in 1992 (almost twenty years after the discovery of charm quark) with the subsequent decay  $h_c \rightarrow J/\psi\pi^0$  [1] :

$$\begin{aligned}\Gamma(h_c) &\sim 800 \text{ keV}, \\ \Gamma(h_c \rightarrow J/\psi + \pi^0) &\sim O(1) \text{ keV}, \\ \Gamma(h_c \rightarrow J/\psi + \pi\pi) &< 0.18 \Gamma(h_c \rightarrow J/\psi + \pi^0).\end{aligned}\tag{1}$$

The data supports Voloshin's approach [2] instead of Kuang-Yan's model [3] as discussed in Refs. [1] and [4]. Another spin-singlet S–wave state  $\eta'_c$  can be reached in the  $e^+e^-$  annihilation via a spin-flip radiative decay  $\psi' \rightarrow \eta'_c + \gamma$ . However, this state has not been confirmed yet. Various potential models predict  $m(\eta'_c)$  to be around  $\approx 3.6$  GeV.

Since BES (Beijing Spectrometer) is planning to search for  $h_c$  and  $\eta'_c$  in the decay channels  $\psi' \rightarrow \eta'_c\gamma$ ,  $h_c\pi^0$  and  $h_c\pi\pi$ , it is worth while to estimate the branching ratios of these decays in the frameworks of QED and QCD multipole expansions, and compare the results with predictions of the Kuang-Yan's model as well as with the data from BES and other experiments. This is precisely the purpose of this report.

First, we discuss  $E1$  and  $M1$  radiative transitions of charmonium states,  $\psi' \rightarrow \chi_{cJ} + \gamma$ ,  $\chi_{cJ} \rightarrow J/\psi + \gamma$  and  $\psi' \rightarrow \eta_c + \gamma$ , in order to extract various matrix elements such as  $\langle 1P|r|nS \rangle$  with  $n = 1, 2$  and  $\langle 1S|r^2|2S \rangle$  from the experimental data. These matrix elements enter various hadronic transitions between charmonium states that will be considered later in this work. We also estimate the branching ratio for  $\psi' \rightarrow \eta'_c + \gamma$ . After discussing radiative transitions of charmonium states, we briefly recapitulate how to describe hadronic transitions between heavy quarkonia in QCD multipole expansion and make a simple approximation which ignores nonlocality in time due to the color-octet intermediate state between subsequent emissions of gluons. One can estimate the size of the color-octet Green's function ( $G_8(m_{\psi'})$ ) from the measured decay rate for  $\psi' \rightarrow J/\psi + \pi\pi$  (or  $\eta$ ). For this purpose, one has to use the matrix element  $\langle 1S|r^2|2S \rangle$  determined from  $\psi' \rightarrow \eta_c + \gamma$ . One can use this  $G_8$  to compute other decays such as  $h_c \rightarrow J/\psi + \pi^0$  (or  $\pi\pi$ ) and  $\psi' \rightarrow h_c + \pi^0$  (or  $\pi\pi$ ). Our results are summarized at the end.

It should be emphasized that one cannot make precise predictions for decay rates

of hadronic transitions between heavy quarkonia because of our ignorance of color confinement in QCD. Any attempts to estimate such quantities are admittedly model dependent. Kuang-Yan's model takes into account the nonlocality of  $G_8$  in time, using the string excitations near the flavor threshold. Our approximation will be opposite to Kuang-Yan's model in the sense that we ignore nonlocality of  $G_8$  in time, and regard  $G_8$  as a constant. Predictions of both approaches are a kind of rough estimates, and the actuality may lie somewhere between two approaches. In this sense, our predictions can be considered as complementary to Kuang-Yan's model.

In order to describe hadronic transitions between heavy quarkonia, we have to know such matrix elements as  $\langle nS|r|1P\rangle$  and  $\langle 1S|r^2|2S\rangle$ . In previous estimations, these quantities were replaced by the radius of the initial or the final quarkonium. However, such estimates can be improved, since these matrix elements are also relevant to electric dipole and spin-flip radiative transitions between heavy quarkonia. In this work, we work in the nonrelativistic quantum mechanics, neglecting the relativistic corrections,  $LS$  coupling and hyperfine splittings. Therefore, the results may be affected by a factor of  $\sim 2$  due to such neglected effects.

Let us first consider electric dipole transitions, whose decay rates are described by

$$\Gamma(E1) = \frac{4}{27}\alpha e_q^2 \omega^3 S_{if} (2J_f + 1) |\langle nS|r|mP\rangle|^2. \quad (2)$$

Here,  $J_f$  is the spin of the final quarkonium, and  $\omega$  is the energy of the emitted photon.  $S_{if} = 3$  for  $h_c(^1P_1) \rightarrow \eta_c + \gamma$  and  $\eta' \rightarrow h_c(^1P_1) + \gamma$ , whereas  $S_{if} = 1$  for other transitions.

Comparing (2) and the measured decay rates of  $\chi_{cJ}(1P) \rightarrow J/\psi + \gamma$  and  $\psi' \rightarrow \chi_{cJ}(1P) + \gamma$ , we get

$$|\langle 1S|r|1P\rangle_\psi| \approx 1.58 \text{ GeV}^{-1} = 0.3 \text{ fm}, \quad (3)$$

$$|\langle 2S|r|1P\rangle_\psi| \approx 1.97 \text{ GeV}^{-1} = 0.4 \text{ fm}, \quad (4)$$

where we have taken the weighted average over  $J = 0, 1, 2$  states. Using  $m(h_c) = 3526$  MeV and taking  $m(\eta'_c) = 3600$  MeV as a tentative value, we get

$$\Gamma(h_c \rightarrow \eta_c + \gamma) \approx 460 \text{ keV}, \quad (5)$$

$$\Gamma(\eta'_c \rightarrow h_c + \gamma) \approx 6.5 \text{ keV}, \quad (6)$$

Since  $|\langle nS|r|1P\rangle_\psi|$ 's with  $n = 1, 2$  are determined from the experimental data on  $\Gamma(\chi_{cJ} \rightarrow J/\psi + \gamma)$  and  $\Gamma(\psi' \rightarrow \chi_{cJ}(1P) + \gamma)$ , our predictions (5) and (6) are independent of specific potential models.

Next, we consider a magnetic dipole radiative transition,  $n^3S_1 \rightarrow m^1S_0 + \gamma$ . The decay rate for such a spin-flip radiative decay is given by

$$\Gamma(\psi' \rightarrow \eta_c + \gamma) = \frac{4}{3} \alpha Q_c^2 \frac{\omega^3}{m_c^2} |\langle 1S | j_0(\omega r/2) | 2S \rangle|^2 \quad (7)$$

$$\approx \frac{4}{3} \alpha Q_c^2 \frac{\omega^3}{m_c^2} |\langle 1S | \frac{\omega^2 r^2}{24} | 2S \rangle|^2. \quad (8)$$

In (7),  $j_0(x)$  is the 0-th order spherical Bessel function,  $j_0(x) = \sin x/x$ . We have used the long-wavelength approximation ( $\omega r \ll 1$ ) in (8). From the measured decay rate  $\Gamma(\psi' \rightarrow \eta_c + \gamma) = 0.78 \pm 0.19$  keV, one can extract

$$|\langle 1S | r^2 | 2S \rangle| \approx 2.55 \text{ GeV}^{-2} \quad (9)$$

which satisfies the bound (20) derived from (19). The main point of this work is the following : one can get the matrix element  $|\langle 1S | r^2 | 2S \rangle|$  from  $\psi' \rightarrow \eta_c + \gamma$ . This matrix element will be used later in order to extract  $G_8$  from  $\psi' \rightarrow J/\psi + \eta$ , as described in the following.

Now, let's consider the possibility of finding  $\eta'_c$  state via a radiative transition  $\psi' \rightarrow \eta'_c + \gamma$ . Assuming  $m(\eta'_c) = 3.60$  GeV, the energy of the emitted photon is  $\omega = 84$  MeV, and the matrix element in (7) can be approximated as 1.  $\Gamma(\psi' \rightarrow \eta'_c + \gamma) \approx 0.94$  keV, or  $3.4 \times 10^{-3}$  in the branching ratio. Thus, one would get  $\sim 340$   $\eta'_c$ 's among  $10^5$   $\psi'$ 's. The subsequent decay of  $\eta'_c$  is mainly hadronic through annihilation into two gluons :  $\Gamma(\eta'_c \rightarrow 2g) \approx 1 - 4$  MeV. The decay rate of the spin-flip radiative transition  $\eta'_c \rightarrow J/\psi + \gamma$  is about 0.26 keV, using the matrix element  $|\langle 1S | r^2 | 2S \rangle|$  which was determined above, (9).

Let us now discuss our main subject, the hadronic transitions between heavy quarkonium states in QCD multipole expansion [2] [3] [5]. The basic formulae can be found in previous works, and we write down the relevant equations only here. The decay rates for spin-flip transitions such as  $1^1P_1 \rightarrow 1^3S_1 + \pi^0$  and  $n^3S_1 \rightarrow 1^1P_1 + \pi^0$  are given by [2]

$$\Gamma(1^1P_1 \rightarrow 1^3S_1 + \pi^0) = \Gamma(n^3S_1 \rightarrow 1^1P_1 + \pi^0) = \frac{1}{2\pi} (A_0 I_{SP})^2 |\vec{p}_\pi|. \quad (10)$$

Here,  $A_0 = 1.7 \times 10^{-3}$  GeV<sup>3</sup> is one-third of the matrix element  $\langle \pi^0 | \pi \alpha_s \mathbf{E}^a(0) \cdot \mathbf{B}^a(0) | 0 \rangle$ , and the matrix element  $I_{SP}$  is defined as

$$I_{SP} = -\frac{2\sqrt{3}}{9m_Q} \langle nS | G_8(E) r + r G_8(E) | 1P \rangle, \quad (11)$$

where  $G_8(E)$  is the Green's functions for the color octet  $Q\bar{Q}$  intermediate states :

$$G_8(E) = \sum_k \frac{|\langle k | \rangle \langle k |}{E_k - E}. \quad (12)$$

To get the absolute decay rate for hadronic transitions between quarkonia, it is imperative to know more about “ $I_{SP}$ ” defined in (11). However, it lies beyond our ability to calculate  $I_{SP}$  from the first principle in QCD, because of our ignorance of the confinement in QCD. Therefore, we have to make some reasonable approximations. We take an opposite limit to the Kuang-Yan's approach, namely that we ignore nonlocality in time coming from the color-octet Green's function,  $G_8(E)$ . We simply assume that  $G_8(E)$  is a constant with dimension of inverse mass as in Ref. [4], since this assumption does not lead to any contradictions to existing data :

$$G_8(E) = G_8 = \text{constant}. \quad (13)$$

Under this assumption, we have

$$I_{SP} = -\frac{4\sqrt{3}}{9m_Q} G_8 \langle nS | r | 1P \rangle, \quad (14)$$

and one can use the informations on the matrix elements on  $r$ , Eqs. (3) and (4).

Taking  $m_c = 1.65$  GeV and using (3) and (4), we get

$$\Gamma(h_c(1P) \rightarrow J/\psi + \pi^0) = 0.095 \left( \frac{1.65}{m_c(\text{GeV})} \right)^2 |G_8(\text{GeV}^{-1})|^2 \text{ keV}, \quad (15)$$

$$\Gamma(\psi' \rightarrow h_c(1P) + \pi^0) = 0.032 \left( \frac{1.65}{m_c(\text{GeV})} \right)^2 |G_8(\text{GeV}^{-1})|^2 \text{ keV}. \quad (16)$$

Note that (15) is smaller than the measured value (1) by an order of magnitude if we assume  $G_8 \sim 1 \text{ GeV}^{-1}$ , as in Ref. [6].

In order to get informations on  $G_8(E)$ , let us consider another hadronic transition,  $\psi' \rightarrow J/\psi + \eta$ , which occur through  $E1 - E1$  and  $E1 - M2$  transitions in QCD multipole expansion [3] [5]. Its amplitude is proportional to the following matrix element

$$I_{SS} = \frac{2}{9} \langle mS | r_i | G(E) | r_i | nS \rangle, \quad (17)$$

which is simplified to the following under the assumption (13) on  $G_8(E)$ ,

$$I_{SS} = \frac{2}{9} G_8 \langle mS | r^2 | nS \rangle. \quad (18)$$

Therefore, once  $\langle 2S | r^2 | 1S \rangle$  is known, the absolute decay rates for these decays can be readily obtained. However, potential model calculation of this matrix element is not available on the contrary to dipole matrix elements  $\langle f | r | i \rangle$  and the mean square radius of a quarkonium,  $\langle i | r^2 | i \rangle$ . In Ref. [4], I have used a quantum mechanical sum rule to derive

$$|\langle 1S | r^2 | nS \rangle|^2 < \frac{4}{m_Q} \frac{|\langle 1S | r^2 | 1S \rangle|}{(E_{nS} - E_{1S})}. \quad (19)$$

From  $|\langle 1S | r^2 | 1S \rangle_\psi| = 4 \text{ GeV}^{-2}$ , the bound (19) was found to be

$$|\langle 1S | r^2 | 2S \rangle_\psi|^2 < 16.5 \text{ GeV}^{-4}. \quad (20)$$

By considering  $\psi' \rightarrow J/\psi + \eta$  and using (20), one obtains a lower bound on  $G_8$  [4] :

$$|G_8|^2 > 10.4 \text{ GeV}^{-2}. \quad (21)$$

Assuming  $G_8(m_{\psi'}) \approx G_8(m_{h_c})$ , we get the following lower bound on  $h_c \rightarrow J/\psi + \pi^0$  from (15) :

$$\Gamma(h_c \rightarrow J/\psi + \pi^0) > 1.0 \text{ keV}, \quad (22)$$

This is consistent with E760 data (1), and moreover, very close to it. Of course, this would be uncertain by a factor of  $\sim 2$ , depending on the choice of  $m_c$ . (In Ref. [4], the mass of  $h_c$  was assumed to be 3.51 GeV, and thus numerical results obtained there are slightly different from the results in the present work.)

One of the main points of this work is that one can actually *improve* the lower bound (22) obtained in Ref. [4], using the estimate on  $\langle 1S | r^2 | 2S \rangle$  obtained from  $\psi' \rightarrow \eta'_c + \gamma$ . From (9) and  $\Gamma(\psi' \rightarrow J/\psi + \eta) = (7.51 \pm 1.41) \text{ keV}$ , one obtains

$$|G_8|^2 \approx 26.4 \text{ GeV}^{-2}. \quad (23)$$

Using this  $|G_8|^2$  in (17) and (18), we finally get

$$\Gamma(h_c \rightarrow J/\psi + \pi^0) \approx 2.5 \text{ keV}, \quad (24)$$

$$\Gamma(\psi' \rightarrow h_c + \pi^0) \approx 0.84 \text{ keV}, \quad (25)$$

Note that our prediction for  $B(\psi' \rightarrow h_c + \pi^0) = 0.3\%$  is very close to the current upper limit, and should be checked in the near future at BEPC. Also, these two predictions in our approach are remarkably in accord with predictions by the Kuang–Yan’s model

with  $\alpha_M \simeq 10\alpha_E$ . However, we have only one gauge coupling (namely,  $\alpha_M = \alpha_E$ ) in our model, and this agreement of two approaches must be regarded accidental.

Another hadronic decay mode,  $1^1P_1 \rightarrow 1^3S_1 + \pi\pi$ , does not receive any contribution from the trace of the energy-momentum tensor in QCD, and is not enhanced over  $1^1P_1 \rightarrow 1^3S_1 + \pi^0$  [2]. (There is no analogous decay  $\psi' \rightarrow h_c + \pi\pi$  because the phase space is not available.) Using the result of Ref. [2], we predict

$$\frac{\Gamma(h_c \rightarrow J/\psi + \pi\pi)}{\Gamma(h_c \rightarrow J/\psi + \pi^0)} \approx \frac{\lambda^2}{30} \approx 0.16, \quad (26)$$

Here,  $\lambda$  measures the gluonic contributions to the energy-momentum of a pion. This is consistent with E760 data, and very close to the current upper limit for  $\lambda \sim 2$ . Therefore,  $h_c \rightarrow J/\psi\pi\pi$  may be observed in the near future.

A remark is in order. As mentioned before, we have made an approximation treating the color-octet Green's function  $G_8(E)$  as a constant, and determined it from the measured decay rate for  $\psi' \rightarrow J/\psi + \eta$ . One can actually check the consistency of this approximation making the following observation. The ratio

$$\frac{\Gamma(h_c \rightarrow J/\psi + \pi^0)}{\Gamma(h_c \rightarrow \eta_c + \gamma)} = 2.1 \times 10^{-4} |G_8(\text{GeV}^{-1})|^2 \quad (27)$$

depends on  $|G_8^2|$ , which can be used as an independent determination of  $|G_8^2|$  from the measurements. Another ratio between  $\Gamma(\psi' \rightarrow h_c + \pi^0)$  and  $\Gamma(\psi' \rightarrow \chi_{cJ}(1P) + \gamma)$  can be useful as well. Thus, our assumption on  $G_8$  is simple, does lead to predictions on various hadronic transitions between heavy quarkonia, and can be checked through the measurement of the ratio (27).

In conclusion, productions and decays of  $\eta'_c$  and  $h_c$  in the  $e^+e^-$  annihilations through  $\psi'$  decays are considered in the framework of QCD multipole expansion, assuming the Green's function for the color octet states is a constant. Using the matrix element  $|\langle 1S | r^2 | 2S \rangle|$  extracted from the spin-flip radiative transition  $\psi' \rightarrow \eta_c + \gamma$ , we could estimate the size of  $|G_8|^2$  by fitting  $\psi' \rightarrow J/\psi + \eta$ . This leads to predictions of absolute decay rates,  $B(\psi' \rightarrow \eta'_c + \gamma) \approx 0.34\%$ ,  $\Gamma(\eta'_c \rightarrow J/\psi + \gamma) \approx 0.26 \text{ keV}$ , and (24), (25), which improves the lower bounds on the decay rates for the same processes obtained in Ref. [4]. We also have suggested that our assumption (13) could be checked by measuring the ratio (27). We hope high statistics experiments at BEPC and other places searching for  $\eta'_c$  and  $h_c$  can test predictions made in this work. Having  $\sim 10^5$   $\psi'$  decays would be enough to get the spin-singlet  $\eta'_c$  and  $h_c$  states and their subsequent decay properties.

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